

This document provides a brief update (as of April 2016) for the numerical solver comparison described in detail in the paper Solving Basis Pursuit: Heuristic Optimality Check and Solver Comparison (D. A. Lorenz, M. E. Pfetsch, and A. M. Tillmann; ACM Transactions on Mathematical Software 41(2), 2015, Art. No. 8. DOI: 10.1145/2689662). Please consult this article regarding the test setup and any further information on the HOC scheme, test instance properties and principles for comparison.

The solver versions used for this update are:

SPGL1 (version 1.8)

<https://www.math.ucdavis.edu/~mpf/spgl1/>

YALL1 (version 1.4)

<http://yall1.blogs.rice.edu/>

ℓ_1 -Homotopy (version 2.0)

<http://users.ece.gatech.edu/sasif/homotopy/>

ℓ_1 -Magic (version 1.11)

<http://statweb.stanford.edu/~candes/l1magic/>

SparseLab (version 2.1)

<https://sparselab.stanford.edu/>

ISAL1 (version 1.0)

<http://wwwopt.mathematik.tu-darmstadt.de/spear/>

STELA (version x)

http://www.nts.tu-darmstadt.de/home_nts/staff_nts/mitarbeiterdetails_32448.en.jsp

CPLEX (version 12.5.1.0)

<http://www.cplex.com>

All numerical experiments were conducted using Matlab R2015a on an Intel(R) Xeon(R) CPU E5-4650 2.70 GHz machine with 500 GB RAM running Ubuntu 14.04.4 LTS.

Note: For this update, we omitted the experiments using SoPlex for brevity; its results are similar to the ones described in the paper. Furthermore, we included a novel solver, STELA, that became available in the time since the paper was published; STELA turned out to be one of the best solvers in our comparisons. The new current versions of the Homotopy and YALL1 implementations also appear to have been significantly improved compared to the older versions used for the paper a few years ago.

Darmstadt, April 2016 – Jan Kuske & Andreas M. Tillmann

Figure 1: HOC improves solution accuracy & speed in many cases, for various solvers.

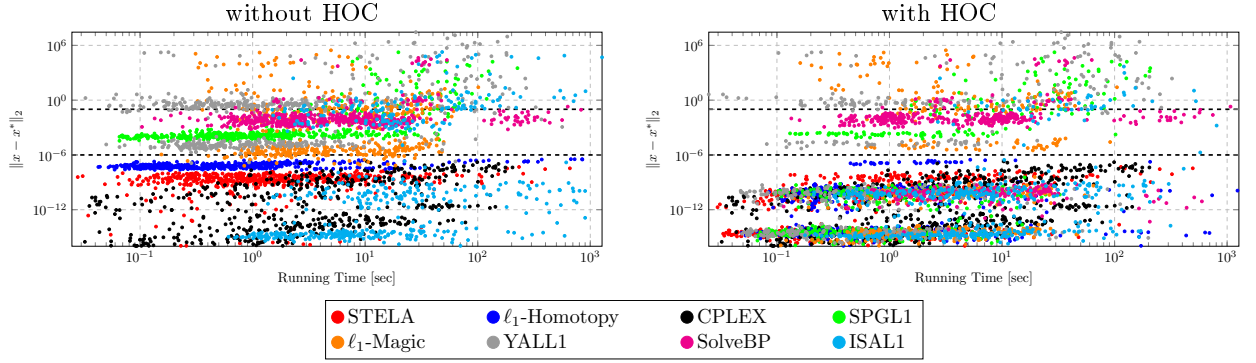


Table 1: Let x be the solution of an instance returned by some solver, and let x^* be the exact solution. We regard an instance as solved if $\|x - x^*\|_2 < 10^{-6}$, acceptable if $10^{-6} \leq \|x - x^*\|_2 < 10^{-1}$ and unsolved if $10^{-1} \leq \|x - x^*\|_2$, respectively.

Solver	HOC	% solved			% acceptable			% unacceptable		
		HDR	LDR	all	HDR	LDR	all	HDR	LDR	all
CPLEX	-	100.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
ℓ_1 -Hom.	woHOC	100.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
	wHOC	100.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
STELA	woHOC	100.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
	wHOC	100.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
SPGL1	woHOC	0.00	0.00	0.00	83.94	90.51	87.23	16.06	9.49	12.77
	wHOC	70.07	71.17	70.62	14.96	19.34	17.15	14.96	9.49	12.23
ℓ_1 -Magic	woHOC	0.00	15.33	7.66	54.01	82.48	68.25	45.99	2.19	24.09
	wHOC	62.04	75.91	68.98	5.11	21.90	13.50	32.85	2.19	17.52
YALL1	woHOC	0.00	1.09	0.55	7.66	79.56	43.61	92.34	19.34	55.84
	wHOC	60.95	63.87	62.41	0.00	17.52	8.76	39.05	18.61	28.83
SolveBP	woHOC	0.00	0.00	0.00	85.04	99.27	92.15	14.96	0.73	7.85
	wHOC	70.44	0.00	35.22	14.60	99.27	56.93	14.96	0.73	7.85
ISAL1	woHOC	44.89	94.89	69.89	28.47	0.36	14.42	26.64	4.74	15.69
	wHOC	82.12	97.81	89.96	5.84	0.73	3.28	12.04	1.46	6.75

Table 2: Improvements achievable by integrating HOC into different solvers.

	ℓ_1 -Hom.	STELA	ISAL1	ℓ_1 -Magic	SPGL1	SolveBP	YALL1
solved faster	262/548	432/548	308/381	42/42	0/0	0/0	2/3
(HDR)	(147/274)	(193/274)	(118/122)	(0/0)	(0/0)	(0/0)	(0/0)
(LDR)	(115/274)	(239/274)	(190/259)	(42/42)	(0/0)	(0/0)	(2/3)
solved Extra	0	0	112	336	387	193	339
(HDR)	(0)	(0)	(103)	(170)	(192)	(193)	(167)
(LDR)	(0)	(0)	(9)	(166)	(195)	(0)	(172)
% imp. ERC	57.3	78.0	94.0	85.0	87.3	45.3	78.5
(HDR)	(64.5)	(62.0)	(97.5)	(78.0)	(88.0)	(90.5)	(78.0)
(LDR)	(50.0)	(94.0)	(90.5)	(92.0)	(86.5)	(0.0)	(79.0)
% imp. non-ERC	22.3	81.1	29.7	25.7	25.7	8.1	18.2
(HDR)	(24.3)	(93.2)	(35.1)	(18.9)	(21.6)	(16.2)	(14.9)
(LDR)	(20.3)	(68.9)	(24.3)	(32.4)	(29.7)	(0.0)	(21.6)
% avg. rel. speed-up	-30.6	29.2	35.8	26.6	9.8	-4.3	9.2
(HDR)	(-39.0)	(10.3)	(54.9)	(20.2)	(-0.8)	(1.5)	(17.0)
(LDR)	(-22.2)	(48.1)	(16.7)	(33.1)	(20.4)	(-10.2)	(1.4)
# faster	262	432	443	462	384	287	370
% speed-up if faster	(38.8)	(53.9)	(65.0)	(37.7)	(44.2)	(35.5)	(48.9)
# slower	286	116	105	86	164	261	178
% overhead if slower	(94.2)	(62.8)	(87.7)	(32.6)	(70.6)	(48.1)	(73.2)

Table 3: Runtime comparison for the three solvers that solved every single instance of our test set.

without HOC						with HOC					
n	Instances		ℓ_1 -Homotopy	STELA	CPLEX	n	Instances		ℓ_1 -Homotopy	STELA	CPLEX
1024	72	avg. t	0.16	0.67	1.56	1024	72	avg. t	0.22	0.24	1.56
		max t	0.70	8.78	9.30			max t	1.15	4.14	9.30
1536	72	avg. t	0.19	1.26	4.79	1536	72	avg. t	0.26	0.59	4.79
		max t	0.52	10.28	50.54			max t	0.72	9.15	50.54
2048	144	avg. t	0.57	4.43	13.34	2048	144	avg. t	0.66	2.68	13.34
		max t	3.26	182.43	119.22			max t	3.41	123.22	119.22
3072	72	avg. t	0.96	8.78	33.74	3072	72	avg. t	1.07	6.31	33.74
		max t	2.89	232.79	203.81			max t	4.05	113.83	203.81
4096	94	avg. t	1.34	18.83	31.34	4096	94	avg. t	1.15	16.00	31.34
		max t	8.07	397.10	176.32			max t	5.99	323.51	176.32
6144	16	avg. t	3.26	0.66	0.08	6144	16	avg. t	2.66	0.45	0.08
		max t	22.35	3.24	0.14			max t	13.69	2.53	0.14
8192	22	avg. t	3.52	46.43	11.36	8192	22	avg. t	2.83	35.12	11.36
		max t	15.73	658.77	129.12			max t	13.17	495.24	129.12
12288	4	avg. t	2.79	2.12	0.15	12288	4	avg. t	2.60	0.99	0.15
		max t	5.90	4.41	0.22			max t	4.80	2.31	0.22
16384	16	avg. t	29.79	1.96	0.68	16384	16	avg. t	37.46	0.95	0.68
		max t	125.44	7.82	2.29			max t	171.78	4.66	2.29
24576	16	avg. t	132.03	10.09	0.63	24576	16	avg. t	139.95	3.97	0.63
		max t	726.47	50.03	1.40			max t	735.77	19.16	1.40
32768	16	avg. t	189.56	15.71	1.51	32768	16	avg. t	186.09	6.82	1.51
		max t	850.40	109.10	4.22			max t	1236.41	47.89	4.22
49152	4	avg. t	143.50	24.93	3.34	49152	4	avg. t	109.40	10.37	3.34
		max t	342.68	72.66	4.38			max t	206.87	27.35	4.38

Please see the reference provided at the beginning of this document for details on how to interpret the following figures and descriptions of the abbreviations.

